Finite Element based optimization of form and material orientation of fiber-reinforced composites

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Hannover Messe 2013 – Werkstoff-Forum
Outline

1. Introduction to Carat++
2. Finite element based structural optimization
3. The composite shell element
4. Fiber-optimization of a single-layer orthotropic plate
5. Combined shape and fiber optimization
   5.1 Navier supported plate
   5.2 Pressure bulkhead
6. Conclusion
1. Introduction to Carat++

Computer Aided Research and Analysis Tool

A flexible finite element software for:

- Simulation:
  - (Non-)Linear Statics and (Non-)Linear Dynamics
  - Eigenfrequency Analysis, Buckling Analysis
  - Contact Analysis
  - Analysis for Reduced Models
  - Controlled Structures
  - Interface to FSI analysis, …

- Formfinding and cutting patterning of membranes

- Isogeometric analysis

- Structural Optimization

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2. Finite element based structural optimization

*Parameterization and design*

**CAGD – Computer Aided Geom. Design**
- Optimization variables:
  - geometry parameters

- time consuming
- limited design space

**CAGD free (= FE-based)**
- Optimization variables:
  - based on FE model:
    - coordinates, thickness, material orientation

- optimization runs out of the box
- maximum design space
- regularization necessary
2. Finite element based structural optimization

Sensitivity analysis: Adjoint semi analytic approach

Programming issues concerning of gradient computation:
• Efficiency
• No design variable specific code

Adjoint formulation of derivatives:

\[
\frac{d\psi}{d\mathbf{s}} = \frac{\partial\psi}{\partial \mathbf{s}} + \frac{\partial\psi}{\partial \mathbf{d}} \frac{d\mathbf{d}}{d\mathbf{s}}
\]

avoid explicit derivatives of primary variables

\[
\frac{d\psi}{d\mathbf{s}} = \frac{\partial\psi}{\partial \mathbf{s}} + \lambda \cdot \frac{\partial \mathbf{h}}{\partial \mathbf{s}}; \text{ with } \lambda = \left( \frac{\partial\psi}{\partial \mathbf{d}} \right)^T \left( \frac{\partial\psi}{\partial \mathbf{d}} \right)^{-1}
\]

Semi-analytical sensitivities of element properties:

\[
\frac{\partial K^e}{\partial s_i} \approx \frac{K(x_i + \Delta s_i) - K}{\Delta s_i} = \frac{\Delta K}{\Delta s_i}
\]

• Low coding effort
• Can be used for any design variable type
2. Finite element based structural optimization

*Gradient filtering based on convolution*

Filter type and size (design decision)

\[
(f * g)(\xi) := \int_{\mathbb{R}^n} f(y)g(\xi - y)\,dy
\]

\[
f, g : \mathbb{R}^n \to \mathbb{C}
\]

Filter function of 3rd order

\[
f(d) = 20 \left( \frac{d^1}{3\pi \cdot r} - 10 \left( \frac{d^2}{\pi \cdot r^2} \right) + 10 \left( \frac{d^3}{3\pi \cdot r^3} \right) \right)
\]

2. Finite element based structural optimization

*Gradient filtering based on convolution*

The filter is not just a mathematical smoothen, it is a design tool!
2. Finite element based structural optimization

Gradient filtering based on convolution

Filter radius: 100

Filter radius: 200

Filter radius: 400
3. The composite shell element

**Single-layer shell formulation:**

- Mainly based on the PhD of Manfred Bischoff at the Universität Stuttgart
- 7-parameter-shell (3x transl., 3x diff. of director, 1x EAS)
  - Thickness changes are considered
- DOFs: 6 x number of nodes
- RM-kinematics
- EAS and ANS improvement
- 2d-geometry, 3d-physics
- Material law is pre-integrated over the thickness:

\[ D_K^{ijkl} = \int_{-1}^{1} (\theta^3)^K \cdot C^{ijkl} \cdot \left(\frac{h}{2}\right)^{K+1} \cdot d\theta^3 \]

- Unmodified three dimensional material laws can be used
3. The composite shell element

**Multi-layer shell formulation:**

- Formulation was extended to a single director multi layer shell:
  \[ \rightarrow \text{still 6 DOFs per node} \]
- Material law and stresses are integrated over the complete thickness

\[
D_{K}^{ijkl} = \sum_{i=1}^{n} \int_{\theta_{i}^{3,\text{lower}}}^{\theta_{i}^{3,\text{upper}}} (\theta^{3})^{K} \cdot C^{ijkl} \cdot \left(\frac{h}{2 \cdot h_{L}}\right)^{K+1} \cdot d\theta^{3}
\]

4. Fiber-optimization of a single-layer orthotropic plate under pressure load

\[ E_\parallel = 44,500 \text{ MPa} \]
\[ E_\perp = 13,000 \text{ MPa} \]
\[ G_{\parallel \parallel} = 5,600 \text{ MPa} \]
\[ G_{\parallel \perp} = 5,100 \text{ MPa} \]
\[ \nu = 0.25 \]
\[ \rho = 2000 \text{ kg/m}^3 \]

Thickness: 10cm

Load: Pressure
4. Fiber-optimization of a single-layer orthotropic plate under pressure load

Optimization of fibre orientations

- 1,600 variables
- Response function: strain energy
- CG-Algorithm
- max. change of angle: 10° per step
- 3-point linesearch
4. Fiber-optimization of a single-layer orthotropic plate under pressure load
5. Combined shape and fiber optimization

5.1 Navier supported plate

- Orthotropic material
- Uniform fiber orientation
- Mixed set of design variables
  - nodal position of all unsupported nodes
  - fiber orientation in each element

Different filters for different design variables:
- linear filter for fiber variables (R=0.8)
- cubic filter for shape variables (R=3.0)
5. Combined shape and fiber optimization

5.1 Navier supported plate
5. Combined shape and fiber optimization

5.1 Navier supported plate

Load carrying pathes of optimal design
5. Combined shape and fiber optimization

5.2 Pressure bulkhead
5. Combined shape and fiber optimization

5.2 Pressure bulkhead

Optimization of shape and fiber orientations while the stack has to remain symmetric.

Objective: strain energy

1 design variable per node
2 design variables per element

Composite structure with symmetric stack: 0 - 90 - 90 - 0
5. Combined shape and fiber optimization

5.2 Pressure bulkhead

Optimization of shape:
Change of geometry
5. Combined shape and fiber optimization

5.2 Pressure bulkhead

Optimization of fiber orientations:
- Outer plies
- Inner plies
5. Combined shape and fiber optimization
5.2 Pressure bulkhead

Buckling load increased by 70%
6. Conclusion

FE-based structural optimization with Carat++

- minimum modeling effort
- large design space (adjoint sensitivity analysis)
- large application field:
  - (non-)linear stiffness, mass, eigenfrequency, buckling,…
- huge models via parallelization (shared and distributed memory)
- abstract definition of design variables allows an easy change of optimization disciplines (nodal coordinates, nodal shell thickness, ply material angle, …)
Finite Element based optimization of form and material orientation of fiber-reinforced composites

Thank you for your attention

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